WEEKLY TEST TYJ-02 MATHEMATICS SOLUTION 11 AUGUST 2019

31. (a) $\tan 1^\circ \tan 2^\circ .... \tan 89^\circ$

$= (\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \tan 88^\circ)...... = 1\times 1\times 1.... = 1.$

32. (a) $\sin\theta + \cos\theta = 1$

Squaring on both sides, we get

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$\therefore \sin\theta\cos\theta = 0$.

33. (b) Since $\text{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$

$\left(\because \tan\theta = -\frac{4}{3}\right)$

$\sin^2\theta = \frac{1}{\text{cosec}^2\theta} = \frac{16}{25} \Rightarrow \sin\theta = \pm\frac{4}{5},$

Both the values are acceptable, since $\tan\theta = -\frac{4}{3}$

i.e., θ lies in 2^{nd} or 4^{th} quadrant.

34. (b) $\sqrt{\left(\frac{1-\sin\theta}{1+\sin\theta}\right)} + \sqrt{\left(\frac{1+\sin\theta}{1-\sin\theta}\right)}$ is the sum of two positive quantities and hence the result must be positive.

But for $\frac{\pi}{2} < \theta < \pi$, we have the sum equal to $\frac{1-\sin\theta+1+\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{2}{\cos\theta}$; which is negative.

($\because$ $\cos\theta$ is negative for θ lying in 2^{nd} quadrant). So the required positive value $= \frac{-2}{\cos\theta} = -2\sec\theta, \left(\frac{\pi}{2} < \theta < \pi\right)$

35. (a) We have $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$\Rightarrow \cos\theta = (\sqrt{2}+1)\sin\theta \Rightarrow (\sqrt{2}-1)\cos\theta = \sin\theta$

$\Rightarrow \sqrt{2}\cos\theta - \cos\theta = \sin\theta \Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta.$

36. (b) $\sec\theta + \tan\theta = p \Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$

Subtracting second from first, we get $2\tan\theta = p - \frac{1}{p}$

$\Rightarrow \tan\theta = \frac{p^2-1}{2p}.$

37. (d) $\tan A + \cot A = 4$

$\Rightarrow \tan^2 A + \cot^2 A + 2\tan A\cot A = 16$

$\Rightarrow \tan^2 A + \cot^2 A = 14 \Rightarrow \tan^4 A + \cot^4 A + 2 = 196$

$\Rightarrow \tan^4 A + \cot^4 A = 194.$

38. (c) We have, $\sin x + \sin^2 x = 1$

or $\sin x = 1 - \sin^2 x$ or $\sin x = \cos^2 x$

$\therefore \cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x - 2$

$= \sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x - 2$

$= (\sin^2 x)^3 + 3(\sin^2 x)^2 \sin x$

$+ 3(\sin^2 x)(\sin x)^2 + (\sin x)^3 - 2$

$= (\sin^2 x + \sin x)^3 - 2 = (1)^3 - 2$

$[\because \sin x + \sin^2 x = 1 (\text{given})]$

= – 1.

39. (a) We know that one of the factor of the given expression is $\cos 90° = 0$.

Therefore $\cos 1°. \cos 2°. \cos 3° ... \cos 179° = 0$

40. (b) Since $\sin 190° = -\sin 10°, \sin 200° = -\sin 20°,$

$\sin 210° = -\sin 30°, \sin 360° = \sin 180° = 0$ etc.

41. (c) $6(\sin^6 \theta + \cos^6 \theta) - 9(\sin^4 \theta + \cos^4 \theta) + 4$

$= 6[(\sin^2 \theta + \cos^2 \theta)^3 - 3\sin^2 \theta \cos^2 \theta (\sin^2 \theta + \cos^2 \theta)]$

$-9[(\sin^2 \theta + \cos^2 \theta)^2 - 2\sin^2 \theta \cos^2] + 4$

$= 6[1 - 3\sin^2 \theta \cos^2 \theta] - 9[1 - 2\sin^2 \theta \cos^2 \theta] + 4$

$= 6 - 9 + 4 = 1$.

42. (a) $x = \cos 40° + \cos 130° = 2\cos 85° \cos 45° > 0$.

43. (b) We have $x = y \cos \frac{2\pi}{3} = z \cos \frac{4\pi}{3}$

$\Rightarrow \frac{x}{1} = \frac{y}{-2} = \frac{z}{-2} = \lambda$ (say) $\Rightarrow x = \lambda, y = -2\lambda, z = -2\lambda$

$\therefore xy + yz + zx = -2\lambda^2 + 4\lambda^2 - 2\lambda^2 = 0$.

44. (c) $\cos A - \sin A = \cos \frac{5\pi}{4} - \sin \frac{5\pi}{4}, \left(\because A = \frac{5\pi}{4}\right)$

$= -\cos \frac{\pi}{4} + \sin \frac{\pi}{4} = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = 0$.

45. (d) We know that $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$= \frac{1}{\sqrt{10}} \sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}} \sqrt{1 - \frac{1}{10}}$

$= \frac{1}{\sqrt{10}} \sqrt{\frac{4}{5}} + \frac{1}{\sqrt{5}} \sqrt{\frac{9}{10}} = \frac{1}{\sqrt{50}}(2 + 3) = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin(A + B) = \sin \frac{\pi}{4}$

Hence, $A + B = \frac{\pi}{4}$.